<u>EW physics below the EW scale</u>

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- Schwartz 29.4

We can finally turn towards the Fermi theory. The goal is to find the leading operators that modify QED and encode the dynamics of the W & Z bosons. The theory will hold for $E \ll m_W, m_Z$.

The relevant Lagrangian is

$$\mathcal{L}_{SM} = m_W^2 |W|^2 + \frac{1}{2} m_Z^2 Z^2 + \frac{g}{2\sqrt{2}} W_\mu^+ J_\mu^- + \frac{g}{2\sqrt{2}} W_\mu^- J_\mu^+ + \frac{g}{c_W} Z_\mu J^{0\mu} + \dots$$

We neglected the boson kinetic terms, since they will lead to extra derivatives. We neglected W/Z self-int, since it will give operators with higher powers of $1/M$.

The EOMs are

$$m_W^2 W_\mu^+ + \frac{g}{2\sqrt{2}} J_\mu^+ = 0 \qquad \qquad W_\mu^+ = -\frac{g}{2\sqrt{2}\, m_W^2} J_\mu^+$$

$$\rightarrow$$

$$m_Z^2 Z_\mu + \frac{g}{c_W} J_\mu^0 = 0 \qquad \qquad Z_\mu = -\frac{g}{c_W m_Z^2} J_\mu^0$$

which leads to

$$\mathcal{L}^{F.T.} = \frac{g^2}{8m_W^2} J^+ \cdot J^- - 2\frac{g^2}{8m_W^2} J^+ \cdot J^-$$

$$+ \frac{g^2}{2c_W^2 m_Z^2} J_0 \cdot J_0 - \frac{g^2}{c_W^2 m_Z^2} J_0 \cdot J_0$$

$$= -\frac{g^2}{8m_W^2} J^+ \cdot J^- - \frac{g^2}{2m_W^2} J_0 \cdot J_0 \qquad (c_W^2 m_Z^2 = m_W^2)$$

Defining

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8m_W^2} = \frac{1}{2v^2}$$

We obtain the EFT

$$\mathcal{L}^{F.T.} = \sum_i \bar{\psi}_i i \not{D} \psi_i - \frac{G_F}{\sqrt{2}} J_\mu^+ J^{-\mu} - \frac{4G_F}{\sqrt{2}} J_\mu^0 J^{0\mu}$$

QED: $\partial_\mu - i q_f e A_\mu$

charged current

weak current

with the currents being those already introduced

$$J^{-\mu} = (J^{+\mu})^{*} = \sum_{\text{fam.}} \left(\bar{\nu}\gamma^{\mu}(1-\gamma^{5})\, e + \bar{u}\gamma^{\mu}(1-\gamma^{5})\, d \right)$$

$$J^{0\mu} = \sum_{i} \bar{\psi}_i \gamma^{\mu} (g_V^i - g_A^i \gamma^5) \psi_i$$

$$\text{with} \quad g_V^i = \frac{1}{4}\left[2\, T_i^3 - 4\, s_W^2\, Q_i\right]$$

$$g_A^i = \frac{1}{4}\left[2\, T_i^3\right]$$

We will study the low energy pheno of these interactions.
While driven by irrelevant operators, they have important effects since they break many symmetries of QED. In particular, they allow for the decay of particles, like the muon and neutron.

As you've seen in the exercise, the muon decay is given by

$$\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) = \frac{G_F^2}{192\pi^3} m_\mu^5$$

which leads to an approximate determination of G_F,

$$G_F = 1.1664 \cdot 10^{-5}\ \text{GeV}^{-2}$$

or,

$$G_F = \frac{1}{\sqrt{2}\, v^2} \quad \Rightarrow \quad v \simeq 246\ \text{GeV}$$

- Neutral current

The Fermi theory interactions

$$\mathcal{L}_{int} = -\frac{G_F}{\sqrt{2}} J_\mu^+ J^{-\mu} - \frac{4G_F}{\sqrt{2}} J_\mu^0 J^{0\mu}$$

consist of a charged current and a neutral current interactions. The Z boson, and therefore the neutral current interactions, were added to the SM after their discovery, much after charged currents.

- While charged currents mediate decays, neutral currents do not, and should be probed by scattering experiments.

We discuss $\mathcal{L}^{n.c.}$ in reactions of the type $\nu e^- \to \nu e^-$, with ν being a neutrino produced in a reactor or in the decay of some other particle. From these we learn that

1) Neutral currents do exist
2) Interactions are given by the SM g_A^i, g_V^i.

- We can write some generalized neutral current

$$\mathcal{L}^{n.c.} = -\frac{4G_F}{\sqrt{2}} J_\mu^{(gen)0} J^{(gen)0\mu}$$

$$; \quad J_\mu^{(gen),0} = \sum_{L=e,\mu} \bar{L} \gamma^\mu (g_V^\ell - g_A^\ell \gamma^5) L + \sum_{L=e,\mu} \bar{\nu}_L \gamma^\mu \left(\frac{1}{4} - \frac{1}{4}\gamma^5\right) \nu_L$$

where the same coupling is assumed for e^-, μ, and ν part is like SM.

We want to check whether

$$g_V^e \overset{?}{=} g_V^e\big|_{SM} = \frac{1}{4}(-1 + 4\bar{s}_W^2)$$

$$g_A^e \overset{?}{=} g_A^e\big|_{SM} = -\frac{1}{4}$$

for some value $\bar{s}_W$ of the weak mixing angle.

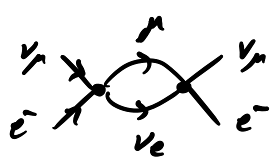
- study the total cross section

$$\text{I}: \ \nu_\mu e^- \longrightarrow \nu_\mu e^-$$

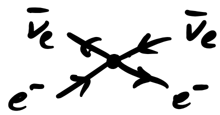
$$\text{II}: \ \bar{\nu}_\mu e^- \longrightarrow \bar{\nu}_\mu e^-$$

$$\text{III}: \ \bar{\nu}_e e^- \longrightarrow \bar{\nu}_e e^-$$

In the absence of neutral currents, process I & II are only loop,



and therefore much suppressed with respect process III, which happens at tree level



Measurement of I, II, III with similar strength is first indication of existence of neutral currents.

- The result is

$$\sigma(\nu_\mu e^- \to \nu_\mu e^-) \equiv \sigma_I = \frac{G_F^2 s}{\pi}\left[(g_V^\ell + g_A^\ell)^2 + \frac{1}{3}(g_V^\ell - g_A^\ell)^2\right]$$

where $s = E_{cm}^2 = 2 m_e E_\nu$

if g's ~ 1, we can compare this cross section with a QED process,

$$\sigma_{QED} \sim \sigma(e^+e^- \to \mu^+\mu^-) \sim \frac{\pi \alpha_{em}^2}{3s}$$

the ratio is given by

$$\frac{\sigma_I}{\sigma_{QED}} \sim \frac{G_F^2 s \; 3s}{\pi^2 \alpha_{em}} \sim 10^2 \; G_F^2 s \sim \left(\frac{\sqrt{s}}{GeV}\right)^4 10^{-6}$$

So at low energies, for $\sqrt{s} \sim GeV$, the correction to QED processes due to the neutral currents is very small.

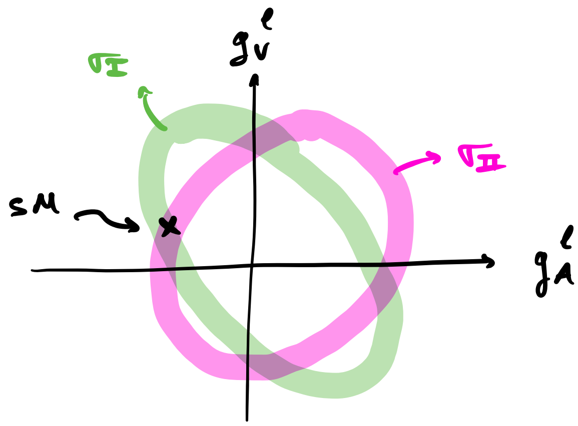
That is why it is seen in processes involving neutrinos, since there is no QED interactions for them.

However,

$$\frac{\sigma_I}{\sigma_{QED}} \sim 1 \quad \text{at} \quad \sqrt{s} \simeq 30\,\text{GeV}$$

- Notice how $\mathcal{L}^{F.T.}$ predicts σ_I to keep growing forever. However, one should remember the validity of the EFT, and above the EW scale the Fermi theory is not an adequate description.

- By measuring $\sigma_I = \bar{\sigma}_I + \delta\sigma_I$, one can draw an ellipse in the g_A^ℓ, g_V^ℓ plane,



where we added

$$\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) \equiv \sigma_{II} = \frac{G_F^2 s}{\pi}\left[(g_V^\ell - g_A^\ell)^2 + \frac{1}{3}(g_V^\ell + g_A^\ell)^2\right]$$

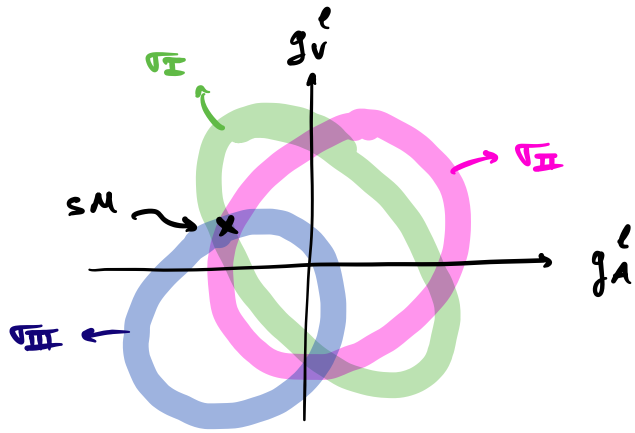
The difference with σ_I is $g_A^\ell \to -g_A^\ell$, so we obtain a reflected ellipse.

σ_I & σ_{II} are invariant under two Z_2 operations: $\begin{cases} g_V^\ell \to -g_V^\ell \\ g_A^\ell \to -g_A^\ell \end{cases}$ $\quad \begin{cases} g_V^\ell \to g_A^\ell \\ g_A^\ell \to g_V^\ell \end{cases}$

The first degeneracy comes from the fact that we are sensitive to only neutral currents, so the amplitude is a linear combination of g_V^ℓ, g_A^ℓ. To resolve it, we should consider processes with charged current contributions, like III

$$\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) \equiv \sigma_{III} = \frac{G_F^2 s}{\pi}\left[(g_V^\ell - g_A^\ell)^2 + \frac{1}{3}(1 + g_V^\ell + g_A^\ell)^2\right]$$

This gives an ellipse centered at $g_V^\ell = g_A^\ell = -1/2$,



Getting rid of the last ambiguity is harder.

Given

$$g_V^\ell - g_A^\ell \gamma^5 = \underbrace{(g_V^\ell + g_A^\ell)}_{\equiv g_L} P_L + \underbrace{(g_V^\ell - g_A^\ell)}_{\equiv g_R} P_R$$

the last Z_2 ambiguity is

$$g_R \to -g_R$$
$$g_L \to g_L$$

so the sign of g_R.

The reason we still have this ambiguity is that the charged current only affects the left-handed fermions. Therefore, we resolve the sign of the left coupling by σ_{III}, but not the one of the right fermions.

There are two options to lift this ambiguity. One is to measure mass effects. This is however hopeless.

The second is to measure processes where QED contributes, like $e^+e^- \to \mu^+\mu^-$.

For the moment, it is enough to say that one can measure the "forward-backward" asymmetry

$$\Delta_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \quad \text{w/} \quad \begin{aligned} \sigma_F &= \text{forward } \mu,\ \theta > \pi/2 \\ \sigma_B &= \text{backward } \mu,\ \theta < \pi/2 \end{aligned}$$

$$\simeq -3 \frac{G_F s}{\sqrt{2}\, e^2} \cdot (2 g_A^\ell)^2$$

This is a measurement of g_A & therefore breaks the degeneracy.

All together leads to

$$\sin^2\theta_W \simeq 0.23$$

see PDG review on Electroweak model and constraints.

12 *10. Electroweak Model and Constraints on New Physics*

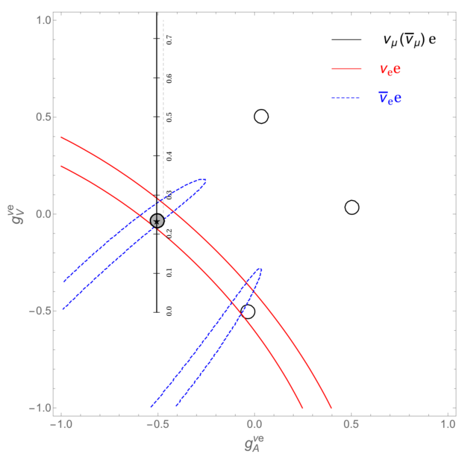


Figure 10.1: Allowed contours in $g_A^{\nu e}$ *vs.* $g_V^{\nu e}$ from neutrino-electron scattering and the SM prediction as a function of $\hat{s}_Z^2$. (The SM best fit value, $\hat{s}_Z^2 = 0.23129$, is also indicated.) The ν_e-e [140,141] and $\bar{\nu}_e$-e [142] constraints are at 1 σ, while each of the four equivalent $\nu_\mu(\bar{\nu}_\mu)$-e [137–139] solutions ($g_{V,A} \to -g_{V,A}$ and $g_{V,A} \to g_{A,V}$) are at the 90% CL. The global best fit region (shaded) almost exactly coincides with the corresponding $\nu_\mu(\bar{\nu}_\mu)$-e region. The solution near $g_A = 0$ and $g_V = -0.5$ is eliminated by $e^+e^- \to \ell^+\ell^-$ data under the weak additional assumption that the neutral current is dominated by the exchange of a single Z boson.